RATIONALITY OF GROWTHS OF GROUPS

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Abstract. For a group G with a finite generating set $S = S^{-1}$, we define $b(n)$ to be the number of group elements that can be written as a product of n elements in S. This is a geometric invariant of G as $b(n)$ is the size of the n-ball in the Cayley graph of G. Gromov's polynomial growth theorem says that any group with $b(n)$ bounded by a polynomial is virtually nilpotent. One then wonders when is $b(n)$ precisely a polynomial, and this turns out to be related to the study of the rational growth of G.

The group G is said to have *rational growth* if the formal power series associated to $b(n)$ is a rational function. It was shown by Cannon that hyperbolic groups have rational growth in any generating set, and the same result was established by Benson for virtually abelian groups. However, Stoll gave a nilpotent group that has rational growth in some, but not all, generating set. This suggests that the rationality may be a measure one can use to decide whether a generating set is easy to compute with.

In this talk, we will discuss the rationality of growths of certain metabelian groups, with the typical example being the Baumslag-Solitar groups. We also generalize past results and study the growths of the higher Baumslag-Solitar groups and torus bundle groups. If time permits, we will also discuss the rationality of the conjugacy growths of groups.

Part of the talk is joint work with Ayla P. Sanchez and with Seongjun Choi and Mark Pengitore.

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